LIBERTY PAPER SET

STD. 10 : Mathematics (Standard) [N-012(E)]

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 10**

Section-A

1. (B) 2 **2.** (A) $\frac{-2}{3}$ **3.** (B) coincident lines **4.** (A) 2 **5.** (C) 78 **6.** (A) $\angle Y = \angle Q$ **7.** C–A–B **8.** $sec\theta$ **9.** Secant **10.** 90° **11.** 1848 **12.** Mode **13.** False **14.** True **15.** False **16.** True **17.** $2\pi r(h + l)$ **18.** $a = -1$ **19.** $\sqrt{2}$ **20.** 6 **21.** (b) Downward open pasabola **22.** (c) Upward open pasabola **23.** (b) $\frac{1}{\sqrt{3}}$ **24.** (c) $\frac{\sqrt{3}}{2}$

Section-B

25. Let us assume that $3 + 2\sqrt{5}$ is rational.

$\therefore 3 + 2\sqrt{5} = \frac{a}{b}$,

(where a and b are coprime & $b \neq 0$)

$\therefore 2\sqrt{5} = \frac{a}{b} - 3 = \frac{a-3b}{b}$

$\therefore \sqrt{5} = \frac{a-3b}{2b}$

Since a and b are integers, we get $\frac{a-3b}{2b}$ is rational and $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

So, our assumption is wrong.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

26. $0.2x + 0.3y = 1.3$

$\therefore 2x + 3y = 13$ (both side multiply by 10) ...(1)

$\therefore x = \frac{13-3y}{2}$ ...(2)

$0.4x + 0.5y = 2.3$

$\therefore 4x + 5y = 23$ (both side multiply by 10) ...(3)

Put value of equation (2) in equation (3),

$4x + 5y = 23$

$\therefore 4\left(\frac{13-3y}{2}\right) + 5y = 23$

$\therefore 26 - 6y + 5y = 23$

$\therefore -6y + 5y = 23 - 26$

$\therefore -y = -3$

$\therefore y = 3$

Put $y = 3$ in equation (2),

$x = \frac{13-3y}{2}$

$\therefore x = \frac{13-3(3)}{2} = \frac{13-9}{2} = \frac{4}{2} = 2$

$\therefore x = 2$

Therefore, the solution is : $x = 2$, $y = 3$.

27. Suppose the one part of 20 is x and the other part is $20 - x$.

Their sum of square is 218

$x^2 + (20 - x)^2 = 218$

$\therefore\ x^2 + 400 - 40x + x^2 - 218 = 0$

$\therefore\ 2x^2 - 40x + 182 = 0$

$\therefore\ x^2 - 20x + 91 = 0$

$\therefore\ x^2 - 13x - 7x + 91 = 0$

$\therefore\ x(x - 13) - 7(x - 13) = 0$

$\therefore\ (x - 13)(x - 7) = 0$

$\therefore\ x - 13 = 0$ OR $x - 7 = 0$

$\therefore\ x = 13$ OR $x = 7$

If the first part $x = 13$, then second part $= 20 - 13 = 7$

or If the first part $x = 7$, then second part $= 20 - 7 = 13$

Hence, the two parts of 20 case 13 and 7.

28. $3x^2 - 2\sqrt{6}x + 2 = 0$

$\therefore\ 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\therefore\ \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$\therefore\ (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$

$\therefore\ \sqrt{3}x - \sqrt{2} = 0$ or $\sqrt{3}x - \sqrt{2} = 0$

$\therefore\ x = \frac{\sqrt{2}}{\sqrt{3}}$ or $x = \frac{\sqrt{2}}{\sqrt{3}}$

$\therefore\ x = \sqrt{\frac{2}{3}}$ or $x = \sqrt{\frac{2}{3}}$

Roots of the equation : $\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$

29. Suppose, $S_{1000} = 1 + 2 + 3 + \dots\dots\dots + 1000$

Now, $S_n = \frac{n}{2}(a + a_n)$

$\therefore\ S_{1000} = \frac{1000}{2}(1 + 1000)$

$\therefore\ S_{1000} = 500 \times 1001$

$\therefore\ S_{1000} = 500500$

So, the sum of the first 1000 positive integers is 500500.

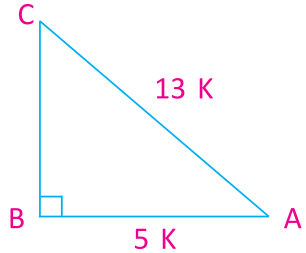
30. Suppose, in ΔABC, $\angle B = 90$

$\cos A = \frac{5}{13}$

$\therefore\ \frac{AB}{AC} = \frac{5}{13}$

$\therefore\ \frac{AB}{5} = \frac{AC}{13} = k \qquad k \neq 0$

$AB = 5k,\ AC = 13k$



According to pythagoras Theoreus,

$AB^2 + BC^2 = AC^2$

$\therefore\ (5k)^2 + BC^2 = (13k)^2$

$\therefore\ 25k^2 + BC^2 = 169k^2$

$\therefore\ BC^2 = 169k^2 - 25k^2$

$\therefore\ BC^2 = 144\ k^2$

$\therefore\ BC = 12k$

Now, $\text{Sin}A = \frac{BC}{AC} = \frac{12k}{13k} = \frac{12}{13}$

$tan A = \frac{BC}{AB} = \frac{12k}{5k} = \frac{12}{5}$

Hence, $\text{Sin}A = \frac{12}{13}$ and $tan A = \frac{12}{5}$

31. LHS = $sec\ A(1 - sin\ A)(sec\ A + tan\ A)$

$= \frac{1}{cos\ A} \cdot (1 - sin\ A) \cdot \left(\frac{1}{cos\ A} + \frac{sin\ A}{cos\ A}\right)$

$= \frac{1}{cos\ A} \cdot (1 - sin\ A) \cdot \left(\frac{1 + sin\ A}{cos\ A}\right)$

$= \frac{(1 - sin\ A)(1 + sin\ A)}{cos\ A \cdot cos\ A}$

$= \frac{1 - sin^2\ A}{cos^2\ A}$

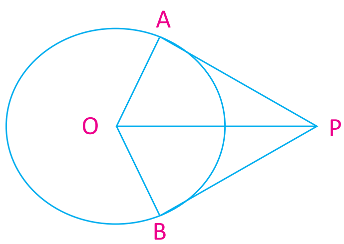
$= \frac{cos^2\ A}{cos^2\ A}$

= 1 = RHS

32. OP is a bisector of $\angle AOB$

$\therefore\ \angle BOP = \frac{\angle AOB}{2} = \frac{80°}{2} = 40°$

$\therefore\ \angle BOP = 40°$



$OB \perp PB$, $\angle OBP = 90°$

In Δ OBP; $\angle BOP + \angle OBP + \angle OPB = 180°$

$\therefore\ 40° + 90° + \angle OPB = 180°$

$\therefore\ \angle OPB = 50°$

33. Diameter of hemisphere = Diameter of cylinder d = 14 cm

$\therefore\ r = 7$ cm

Now, total height = 13 cm

$\therefore$ Height of cylinder + Radius of hemisphere = 13

$\therefore\ h + r = 13$

$\therefore\ h + 7 = 13$

$\therefore\ h = 6$ cm

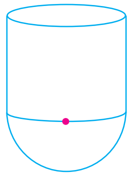
Total inner surface area of the vessel

= CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 7 \times (6 + 7)$



$= 2 \times 22 \times 13$

$= 572\ cm^2$

34. **Mode :**

Here, maximum class frequency is 10 which belongs to modal class 30-35.

$\therefore l$ = lower class limit of modal class = 30

h = class size = 5

f_1 = frequency of modal class = 10

f_0 = frequency of class preceding the modal class = 9

f_2 = frequency of class succeeding modal class = 3

$$\text{Mode } Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\therefore Z = 30 + \left(\frac{10-9}{2(10)-9-3}\right) \times 5$$

$$\therefore Z = 30 + \frac{1 \times 5}{8}$$

$\therefore Z = 30 + 0.625$

$\therefore Z = 30.625$

$\therefore Z = 30.63$

35.

Literacy rate (class)	Number of cities (f_i)	x_i	u_i	f_iu_i
45 – 55	3	50	–2	–6
55 – 65	10	60	–1	–10
65 – 75	11	70 = a	0	0
75 – 85	8	80	1	8
85 – 95	3	90	2	6
Total	$\Sigma f_i = 35$	–	–	$-2 = \Sigma f_iu_i$

$$\text{Mean } \bar{x} = a + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h$$

$$\therefore \bar{x} = 70 + \frac{-2}{35} \times 10$$

$$\therefore \bar{x} = 70 - \frac{4}{7}$$

$\therefore \bar{x} = 70 - 0.57$

$\bar{x} = 69.43$

So, mean literacy rate is 69.43%.

36. One shirt is drawn at random from the carton of 100 shirts. Therefore, there are 100 equally likely outcomes.

(i) The number of outcomes favourable (i.e., acceptable) to Jimmy = 88

Therefore,

$$\therefore \text{P (shirt is acceptable to Jimmy)} = \frac{88}{100}$$

$= 0.88$

(ii) The number of outcomes favourable to Sujatha = 88 + 8 = 96

So, P (shirt is acceptable to Sujatha)

$$= \frac{96}{100} = 0.96$$

37. Here, total number of cards = 52

(i) Suppose A be the event the selected card is not a king.

Number of kings card = 4

$\therefore$ Number of without kings cards = 52 – 4 = 48

$\therefore$ The number of outcomes favourable to A = 48

$\therefore$ P (A) = $\frac{48}{52} = \frac{12}{13}$

(ii) Suppose B be the event the selected card is queen of black colour.

Number of the queen of black colour = 2

$\therefore$ The number of outcomes favourable to B = 2

$\therefore$ P (B) = $\frac{2}{52} = \frac{1}{26}$

Section-C

38. Let $P(x) = 3x^2 - 14x + 5$

then compare with $P(x) = ax^2 + bx + c$

$\therefore a = 3, b = -14, c = 5$

$\alpha + \beta = \frac{-b}{a} = \frac{-(-14)}{3} = \frac{14}{3}$

$\alpha . \beta = \frac{c}{a} = \frac{5}{3}$

(i) $\alpha^2 + \beta^2 = \alpha^2 + 2\alpha\beta + \beta^2 - 2\alpha\beta$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(\frac{14}{3}\right)^2 - 2\left(\frac{5}{3}\right)$

$= \frac{196}{9} - \frac{10}{3}$

$= -\frac{196 - 30}{9}$

$\therefore \alpha^2 + \beta^2 = \frac{166}{9}$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{\frac{14}{3}}{\frac{5}{3}}$

$= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{14}{5}$

39. $x^2 - 2x - 8 = 0$

$\therefore x^2 - 4x + 2x - 8 = 0$

$\therefore x(x - 4) + 2(x - 4) = 0$

$\therefore (x - 4)(x + 2) = 0$

$\therefore x - 4 = 0$ and $x + 2 = 0$

$\therefore x = 4$ and $x = -2$

Let $\alpha = 4$ and $\beta = -2$

Now, Sum of zeros = $\alpha + \beta = 4 + (-2) = 4 - 2 = 2$

and product zeros = $\alpha . \beta = (4)(-2) = -8$

40. Here, AP : 10, 7, 4, ..., –62

$a = 10$, $d = 7 - 10 = -3$, $a_n = -62$

Now, $a_n = a + (n - 1)d$

$\therefore -62 = 10 + (n - 1)(-3)$

$\therefore -62 - 10 = (n - 1)(-3)$

$\therefore \frac{-72}{-3} = n - 1$

$\therefore n - 1 = 24$

$\therefore n = 25$

So, there are 25 terms in the given AP.

$\therefore a_{15} = a + 14d$

$\therefore a_{15} = 10 + 14(-3)$

$\therefore a_{15} = 10 - 42$

$\therefore a_{15} = -32$

41. $a = 17$, $a_n = l = 350$, $d = 9$, $n =$ ____, $S_n =$ ____

$a_n = a + (n - 1)\, d$

$\therefore 350 = 17 + (n - 1)\, 9$

$\therefore 350 - 17 = (n - 1)\, 9$

$\therefore \frac{333}{9} = n - 1$

$\therefore n - 1 = 37$

$\therefore n = 38$

Now, $S_n = \frac{n}{2}(a + a_n)$

$\therefore S_{38} = \frac{38}{2}(17 + 350)$

$= 19\,(367)$

$\therefore S_{38} = 6973$

42. Let P and Q be the points of trisection of AB i.e., AP = PQ = QB

$\therefore$ Therefore, P divides AB internally in the ratio 1:2

$\therefore$ The coordinates of P = $\left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right) = (-1, 0)$

Same as, Q also divides AB internally in the ratio 2:1.

$\therefore$ The coordinates of Q = $\left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right) = (-4, 2)$

Therefore, the coordinates of the points of trisection of the line segment joining A and B are (–1, 0) and (– 4, 2).

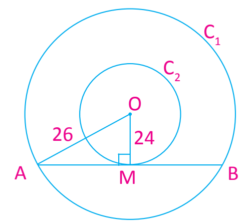
43. The radii of the two O concentric circles are C_1 and C_2.

Radius of C_1 = OA = r_1 = 26 and

Radius of C_2 = OM = r_2 = 24.

The chord AB of C_1 touches C_2 at the Point M.

In Δ OMA; $\angle M = 90°$



$\therefore$ AM = $\sqrt{OA^2 - OM^2}$

$\therefore$ AM = $\sqrt{(26)^2 - (24)^2}$

$\therefore$ AM = $\sqrt{676 - 576}$

$\therefore$ AM = $\sqrt{100}$

$\therefore$ AM = 10

But, AB = 2 AM

$\therefore$ AB = 2×10

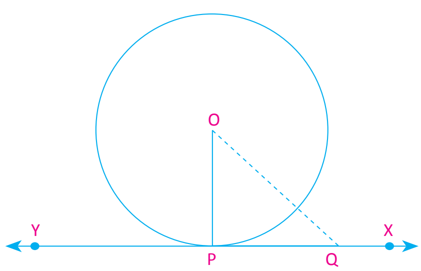
$\therefore$ AB = 20

Thus, the length of chord 20.

44. Given : A circle with centre O and a tangent XY to the circle at a point P.

Prove that : OP is perpendicular to XY. i.e. OP $\perp$ XY

Figure :



Proof : Take a point Q on XY other than P and join OQ.

The point Q must lie out side the circle. If Q lies inside the circle, XY will become a secant and not a tangent to the circle.

Therefore, Q is longer than the radius OP of the circle.

i.e., OQ > OP

Since, this happens for every point on line XY except the point P, OP is the shortest of all the distance of the point O to the point of XY.

So, OP is perpendicular to XY.

45. Here r = 6 cm and $\theta = 60°$

(i) Length of arc = $\frac{\pi r\theta}{180} = \frac{22 \times \overset{2}{\cancel{6}} \times \cancel{60}}{7 \times \cancel{180}} = \frac{44}{7}$ cm

(ii) Area of minor sector = $\frac{\pi r^2\theta}{360} = \frac{\overset{3}{\cancel{22}} \times 6 \times \cancel{6} \times \cancel{60}}{7 \times \underset{\cancel{6}}{\cancel{360}}}$

$= \frac{132}{7}$ cm^2

46. Total number of coins in a piggy bank = 100 + 50 + 20 + 10 = 180

$\therefore$ Total number of outcomes = 180

(i) Suppose event A is the fallen coin will be a 50 p coin.

$\therefore$ P(A) = $\frac{\text{Number of 50 p coin}}{\text{Total number of outcomes}}$

$\therefore$ P(A) = $\frac{100}{180}$

$\therefore$ P(A) = $\frac{5}{9}$

(ii) Suppose event B is the fallen coin will not be. a ₹ 5 coin, therefore event $\overline{B}$ is the fallen coin will be ₹ 5.

$$\therefore P(\overline{B}) = \frac{\text{Number of ₹ 5 coins}}{\text{Total number of outcomes}}$$

$$\therefore P(\overline{B}) = \frac{10}{180}$$

$$\therefore P(\overline{B}) = \frac{1}{18}$$

Now, $P(B) = 1 - P(\overline{B}) = 1 - \frac{1}{18} = \frac{17}{18}$

$$\therefore P(B) = \frac{17}{18}$$

(ii) Suppose event C is the fallen coin will not be a ₹ 1 coin.

$$\therefore P(C) = \frac{\text{Number of ₹ 1 coins}}{\text{Total number of outcomes}}$$

$$\therefore P(C) = \frac{50}{180}$$

$$\therefore P(C) = \frac{5}{18}$$

Section-D

47. Suppose Indian team scored x runs and Sri Lankan's team scored y runs.

Here the sum of $\frac{1}{3}$ shase of India's and $\frac{1}{7}$ shase of Sri Lanka's runs is 137 runs.

$\therefore \frac{1}{3}x \times \frac{1}{7} y = 137$

$\therefore 7x + 3y = 2877$ ....(1)

Here, the indian team won against Sri Lanka, by the minimum runs required, That is wn by 1 run.

$x - y = 1$ ....(2)

Multiply equation (2) by 3 and add in equation (1)

$$\begin{aligned} 7x + 3y &= 2877 \\ 3x - 3y &= 3 \\ \hline 10x &= 2880 \end{aligned}$$

$\therefore x = \frac{2880}{10}$

$\therefore x = 288$

From (2)

$288 - y = 1$

$\therefore 288 - 1 = y$

$\therefore y = 287$

Hence, the Indian team scored 288 runs and the Sri Lankan team scored 287 runs.

48. Suppose the price of 1 kg tea is x and sagar is y Rs. respectivaly.

Let Price of 1 kg of tea is seven times the price of 1 kg sugar.

$\therefore x = 7y$

$\therefore x - 7y = 0$ ...(1)

Let the price of 5 kg of sugar and 2 kg of tea is Rs. 380.

$\therefore 2x + 5y = 380$ ...(2)

Multiply equation (1) by 2 and subtract equation (2)

$$\begin{aligned} 2x - 14y &= 0 \\ 2x + 5y &= 380 \\ - \quad - \quad &\quad - \\ \hline -19y &= -380 \end{aligned}$$

$\therefore y = \frac{-380}{-19}$

$\therefore y = 20$

From (1)

$x - 7\ (20) = 0$

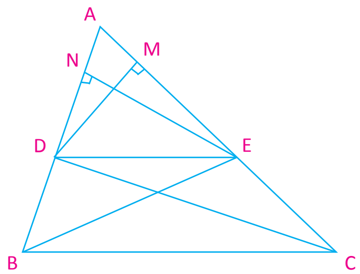
$\therefore x - 140 = 0$

$\therefore x = 140$

Hence, the price of 1 kg tea is 140 Rs. and the price of 1 kg sugar is 20 Rs.

49. Given: In ABC, a line parallel to side BC intersects AB and AC at D and E respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$



Proof : Join BE and CD and also draw DM $\perp$ AC and EN $\perp$ AB.

Then, ADE $= \frac{1}{2} \times AD \times EN$,

BDE $= \frac{1}{2} \times DB \times EN$,

ADE $= \frac{1}{2} \times AE \times DM$ and

DEC $= \frac{1}{2} \times EC \times DM$.

$$\therefore \frac{ADE}{BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad ...(1)$$

$$\text{and } \frac{ADE}{DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad ...(2)$$

Now, Δ BDE and Δ DEC are triangles on the same base DE and between the parallel BC and DE.

then, BDE = DEC ...(3)

Hence from eq^n. (1), (2) and (3),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

50. Fill in the blank given in the proof of the question below. If $\Delta ABE \cong \Delta ACD$ in the given figure, prove that $\Delta ADE \sim \Delta ABC$

Given: $\Delta ABE \cong \Delta ACD$

To Prove: $\Delta ADE \simeq \Delta ABC$

Proof : Here $\Delta ABE \cong \Delta ACD$ (Given) .

$\therefore$ AB = AC and AE = <u>AD</u> (CPCT)

$\therefore \dfrac{AE}{AC} = \dfrac{AD}{AB}$

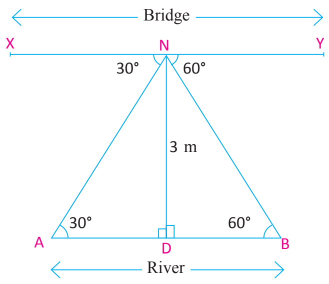
Now in ΔADE and ΔABC,

we have, $\dfrac{AE}{AC} = \dfrac{AD}{AB}$

$\angle DAE = \angle BAC$ [Same (common) angle]

SAS According to condition, $\Delta ADE \sim \Delta ABC$

51.



$\angle XNA = \angle NDA = 30°$

$\angle YNB = \angle NBD = 60°$

DN = 3 m

Here In ΔNDA,

$\tan 30° = \dfrac{ND}{AD}$

$\therefore \dfrac{1}{\sqrt{3}} = \dfrac{3}{AD}$

$\therefore AD = 3\sqrt{3}$ m

In ΔNDB,

$\therefore \tan 60° = \dfrac{ND}{DB}$

$\therefore \sqrt{3} = \dfrac{3}{DB}$

$\therefore DB = \dfrac{3}{\sqrt{3}}$

$\therefore DB = \sqrt{3}$ m

Width of river

AB = AD + DB

$\therefore AB = 3\sqrt{3} + \sqrt{3}$

$\therefore AB = 4\sqrt{3}$ m

$\therefore$ width of river is $4\sqrt{3}$ m

52. Let be the radii of sphereis r.

the surface area of sphere = 1256 cm^2

$\therefore 4\pi r^2 = 1256$

$\therefore 4 \times 3.14 \times r^2 = 1256$

$\therefore r^2 = \dfrac{\overset{4}{\cancel{1256}} \times 100}{4 \times \cancel{314}}$

$\therefore r^2 = 100$

$\therefore r = 10$ cm

Now, the volume of spere = $\frac{4}{3}\ \pi r^3$

$= \frac{4}{3} \times 3.14 \times 10 \times 10 \times 10$

$= \frac{4}{3} \times \frac{314}{100} \times 10 \times 10 \times 10$

$= \frac{12560}{3}$

$= 4186.67\ cm^3$

Hence, the volume of the sphere = $4186.67\ cm^3$

53.

Cylinder	Cone
$h = 2.1$ m	$r = 2$ m
$d = 4$ m	$l = 2.8$ m

$r = 2$ m

The area of convas = CSA of cylinder + CSA of cone

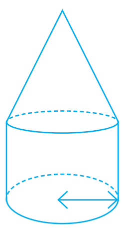
$= 2\pi rh + \pi rl$

$= \pi r\ (2h + l)$

$= \frac{22}{7} \times 2 \times [2(2.1) + 2.8]$

$= \frac{44}{7} \times 7$

$= 44\ m^2$



Now, Cost of canvas = Area of canvas × Cost of canvas

$= 44\ m^2 \times$ ₹ 500 per m^2

= ₹ 22,000

So, it will cost ₹ 22,000 for making such a tent.

54.

Length (in mm)	class	Number of leaves (f_i)	cf
118 – 126	117.5 – 126.5	3	3
127 – 135	126.5 – 135.5	5	8
136 – 144	135.5 – 144.5	9	17
145 – 153	144.5 – 153.5	12	29
154 – 162	153.5 – 162.5	5	34
163 – 171	162.5 – 171.5	4	38
172 – 180	171.5 – 180.5	2	40
Total	–	40	–

Here, $n = 40$

$\therefore \frac{n}{2} = \frac{40}{2} = 20$

The cumulative version 20 immediately after 29 is included in the observation class 144.5 – 153.5, so the median class is 144.5 – 153.5.

l = lower limit of median class = 144.5

cf = cumulative frequency of class preceding the median class = 17

f = frequency of median class = 12

h = class size = 9

Median $M = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$\therefore M = 144.5 + \left(\frac{20 - 17}{12}\right) \times 9$

$\therefore M = 144.5 + \frac{3 \times 9}{12}$

$\therefore M = 144.5 + 2.25$

$\therefore M = 146.75$

Thus, median length of leaves is 146.75 mm.